Electrical Engineering Research Laboratory
The University of Texas

Report No. 68 — 6 May 1953

Comparison of Power Spectrum Estimates of Overwater Microwave Radio Signal and Associate Water Waves

Prepared Under Office of Naval Research Contract Nonr 375(01)

ELECTRICAL ENGINEERING RESEARCH LABORATORY

THE UNIVERSITY OF TEXAS

Report No. 68

6 May 1953

COMPARISON OF POWER SPECTRUM ESTIMATES OF OVERWATER MICROWAVE RADIO SIGNAL AND ASSOCIATE WATER WAVES

by

Wm. J. McKune
H. W. Smith

Prepared Under Office of Naval Research Contract Nonr 375(01)

TABLE OF CONTENTS

FIGURES

FIGURES (Continued)

ABSTRACT

Preliminary investigations of power spectrum estimates for signal strength and corresponding sea-state time fluctuations are reported for an overwater path between two drilling platforms in the Gulf of Mexico.

Signal strength measurements at wavelengths of 9.0, 5.3, 3.2 and 0.86 cms and corresponding ocean-wave recordings are included in the analysis.

The method of analysis is described with a discussion of important limitations and assumption involved in the techniques.

I. INTRODUCTION

A recent report from this Laboratory [1] gave the results of the reflection characteristics of centimeter and millimeter radio waves for a path over the Gulf of Mexico. This report gave these characteristics for wavelengths of 9.0, 5.3, 3.2 and 0.86 cms over an open-sea path between two drilling platforms off Grand Island, Louisiana, (Figure 1).

It is the purpose of this report to present the power spectra estimates of the time fluctuations of these radio signals and the corresponding ocean-wave recordings.

II. DATA INFORMATION

The details of the radio transmitters, receivers and ocean-wave recorder are described in Report 64 [1] and will not be repeated here. The transmitters were located on the seaward drilling platform at heights of either 15 ft or 38 ft msl. The receivers were on the shoreward platform at heights of either 14 ft or 53 ft msl. The platforms were 5027 ft apart and 8 miles off shore. Measurements were made with the antennas both vertically and horizonally polarized.

A record of water-wave height was taken with a step type wave gage, with steps every 0.2 ft. The gage was located near a 14-in. diameter pile on the seaward side of the receiver platform.

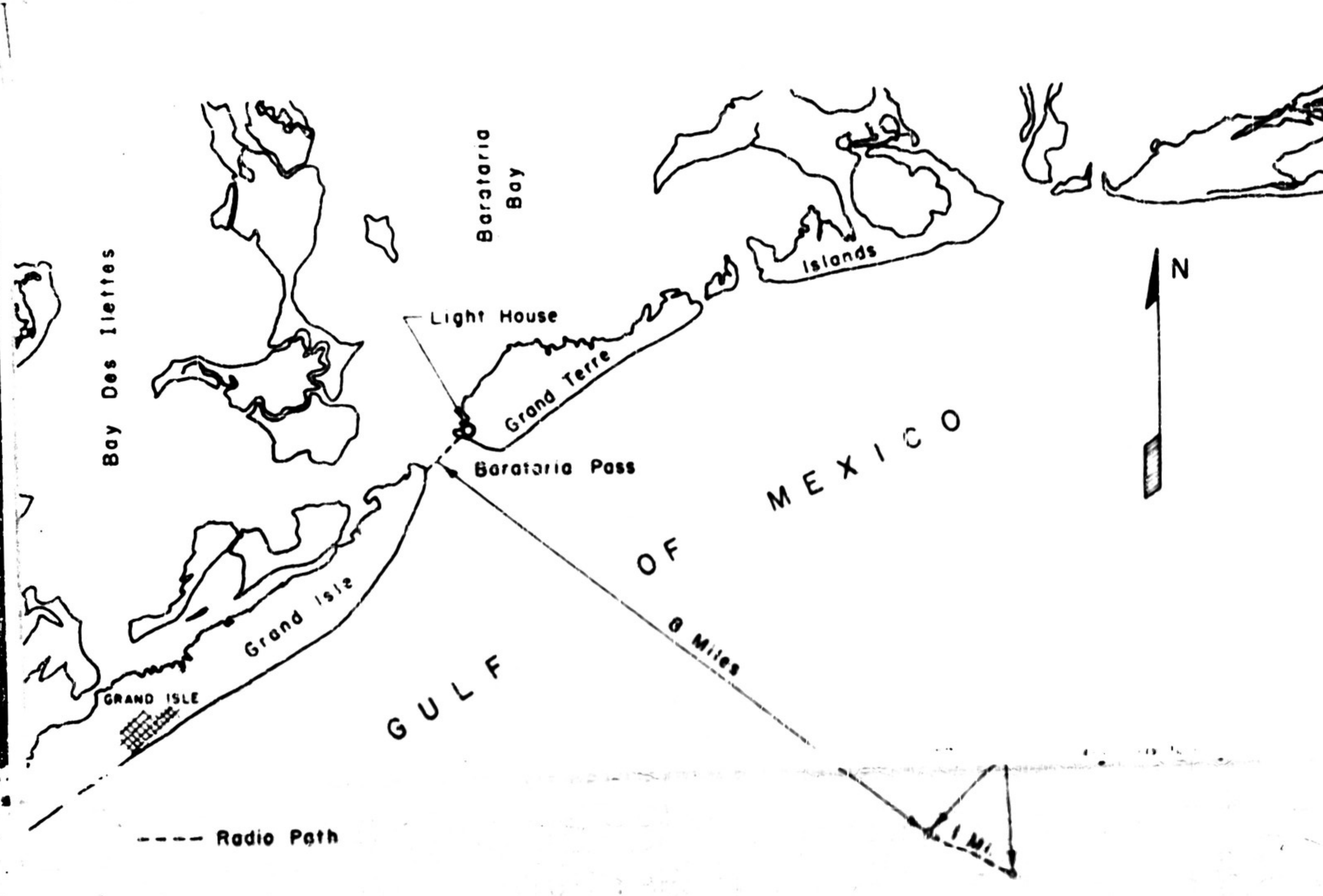


FIG. 1 – LOUISIANA COASTAL REGION NEAR GRAND ISLE

III. WEATHER CO [illegible]

During the two [illegible]ys of the tests (6 and 7 of August) rain showers were common. On 6 August the nor[illegible] southeasterly flow produced relatively low amplitude, long period swells. At 1[illegible]00, on 6 August, a squall line moved into the area from the west and pro[illegible]uced w[illegible]d gusts up to 38 mph at the receiver platform. The wind remained out of the [illegible]st through 7 August with an average speed of 15 mph. The ocean surface respo[illegible]d with wind waves averaging 18 to 24 in. high with occasional waves of 30 in. at [illegible] receiver platform.

IV. DATA SAMPLES

Samples of the [illegible]iginal data taken between the platforms on 6 and 7 August are shown on Figure [illegible] It should be pointed out that these samples were not taken simultaneously and [illegible]t the water data are merely typical of the sea state during the series of runs [illegible]resenting a time span of approximately an hour and a half. However, analyses o[illegible] the data for each run at a given wavelength were made with the signal strength [illegible]d the sea state data taken at the same time.

V. LIMITATION ON T[illegible] DATA

a. Simultaneou[illegible] time records were made of the radio signals and wave heights for intervals of 2½ [illegible]utes. It is recognized that this short time limits the accuracy of the re[illegible]s. Effects of this limited sample of data are discussed in the Appendix.

b. The radio s[illegible]als were received and recorded with a calibration which was approximately li[illegible]r in decibels. The proper interpretation of a comparison of the signal streng[illegible], which is expressed as a logarithmic ratio, and the water height, which is exp[illegible]ssed as a difference, is open to question.

c. The locatio[illegible] of the wave recorder raises the question as to whether the data were representa[illegible]re of the sea state over the path. The question arises as to whether the wave [illegible]ght taken at a single point is a good representation of sea surface. The p[illegible]uity of the pilings of the drilling platform to the wave recorder could be ex[illegible]ted to affect the high-frequency components of the water data.

d. The high-fr[illegible]ency components of the power spectra are further subject to error because of [illegible] limited frequency response of the EA recorders at frequencies above one c[illegible]le per second.

The value of th[illegible] results would then hinge upon the following assumptions with regard to the d[illegible]a.

1. The 2½ [illegible]nutes of data were sufficient to insure that the record approximate[illegible] a statio[illegible]ary time series.

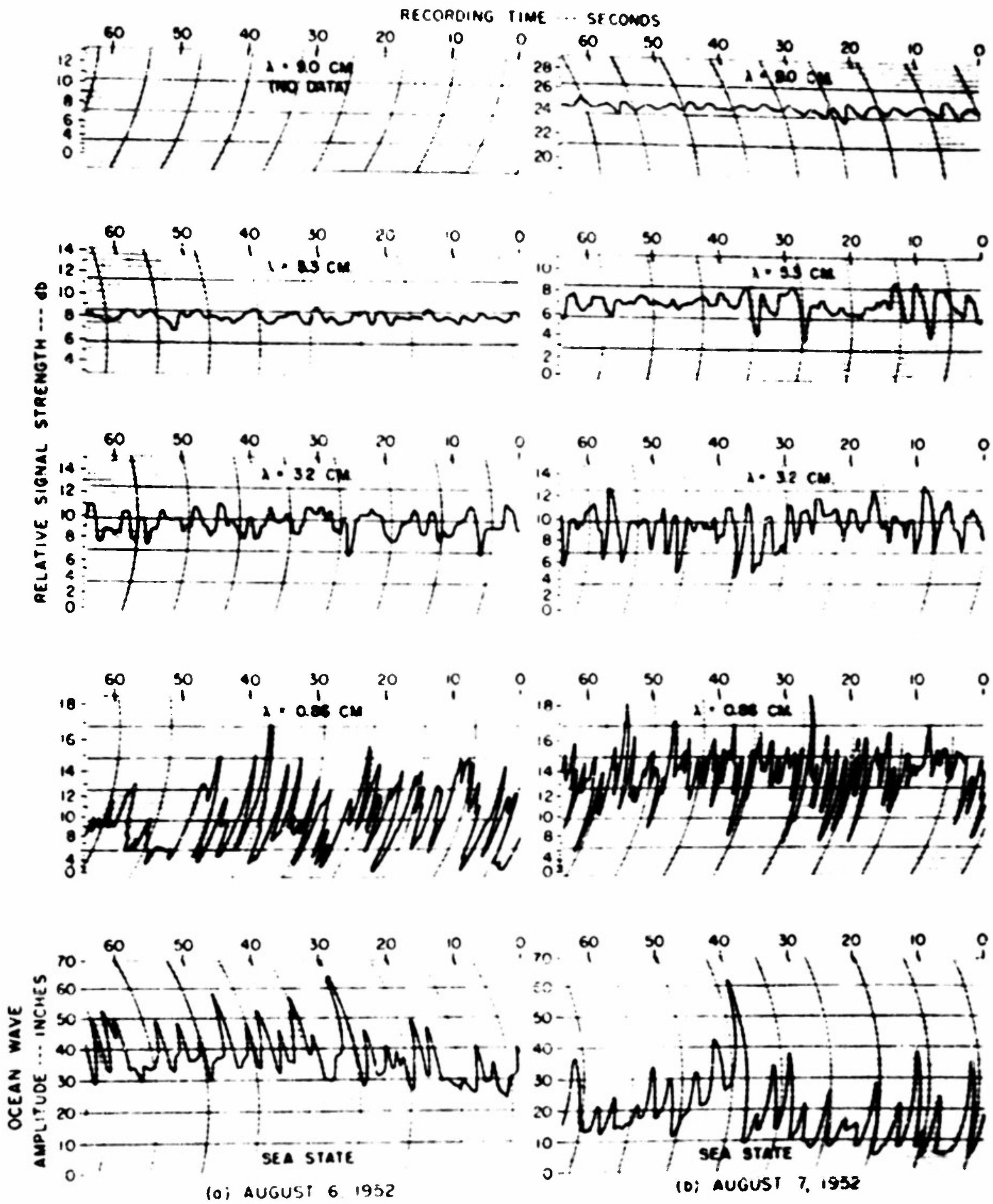


FIG 2 - TIME VARIATIONS IN HORIZONTALLY POLARIZED RADIO SIGNALS BETWEEN PLATFORMS WITH OCEAN WAVE RECORDINGS

2. The amplitudes of the water waves for frequencies above one cycle per second are negligible.

3. It would appear that it would be desirable to compare the sea state with the signal strength in volts, rather than in decibels so that each of the quantities would be expressed as a difference of two quantities. The labor of replotting the original data to a voltage scale and the errors accumulated in such a process prohibited this being done. Hence it is assumed that the frequency components of the signal strength expressed in volts would be the same as when expressed in decibels. This has been checked for one sample of data involving the 3.2 cm signal (see Figure 9), where this was found to be a reasonable assumption.

VI. ANALYSIS OF THE DATA

The power spectra of the data were made by first obtaining their autocorrelation function with the correlation computer developed at this Laboratory [3]. This function was analyzed into its frequency components by a method presented by Tukey [2] and outlined in the Appendix. This results in the power spectra representing the average power in a band about the frequency in question.

For the comparison of the power spectra of the signal strength and simultaneous sea state, curves are plotted on a normalized basis (Figures 3-5, 8-11). In order to show the relative power of the signal strength variations for the various wavelengths, the power spectra of these variations are plotted in proportion to their rms value (Figures 6, 12). A similar set of curves shows the relative power for the sea state variations (Figures 7, 13).

Since it is of interest to know as much as possible about the amplitude distribution of the time variations when dealing with correlation and power spectrum analysis, the signal strength on a decibel basis was plotted on normal distribution coordinates, as shown in Figure 14, for the data of 7 August. This was also done for the sea-state data on the same day as shown in Figure 15. All of the power spectrum estimates shown were for vertically polarized radio signals.

VII. RESULTS

Figures 3, 4 and 5 show the normalized power spectra at the radio signals and simultaneous sea state for wavelengths of 0.86 cm, 3.2 cm and 5.3 cm taken on 6 August. No analysis of the 9-cm signal was made, since the signal had no appreciable variation. Figures 6 and 7 show the relative power spectra of these radio signals and sea states.

Figures 8, 9, 10 and 11 show the normalized power spectra at 0.86-, 3.2-, 5.3- and 9-cm radio signals and corresponding sea states for 7 August. Figures 12 and 13 show the relative spectra of this data. Figure 9 also shows the results of the analysis of the radio signal on a voltage basis as well as on a decibel basis.

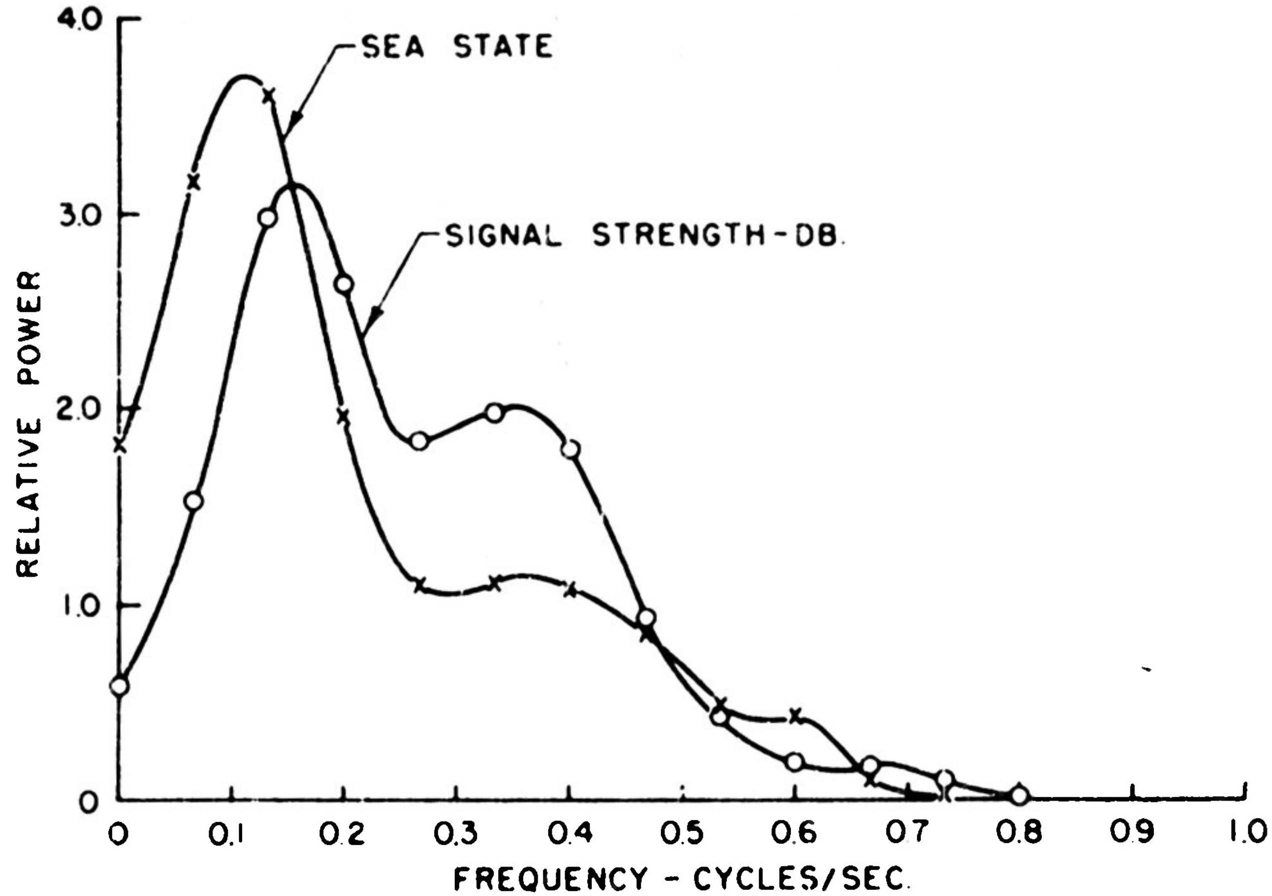


FIG. 3-NORMALIZED POWER SPECTRUM ESTIMATES (TUKEY METHOD) FOR 5.3 CM. RADIO SIGNALS AND SIMULTANEOUS SEA STATE - AUGUST 6, 1952, TIME: 0820

EERL 1349
4-53

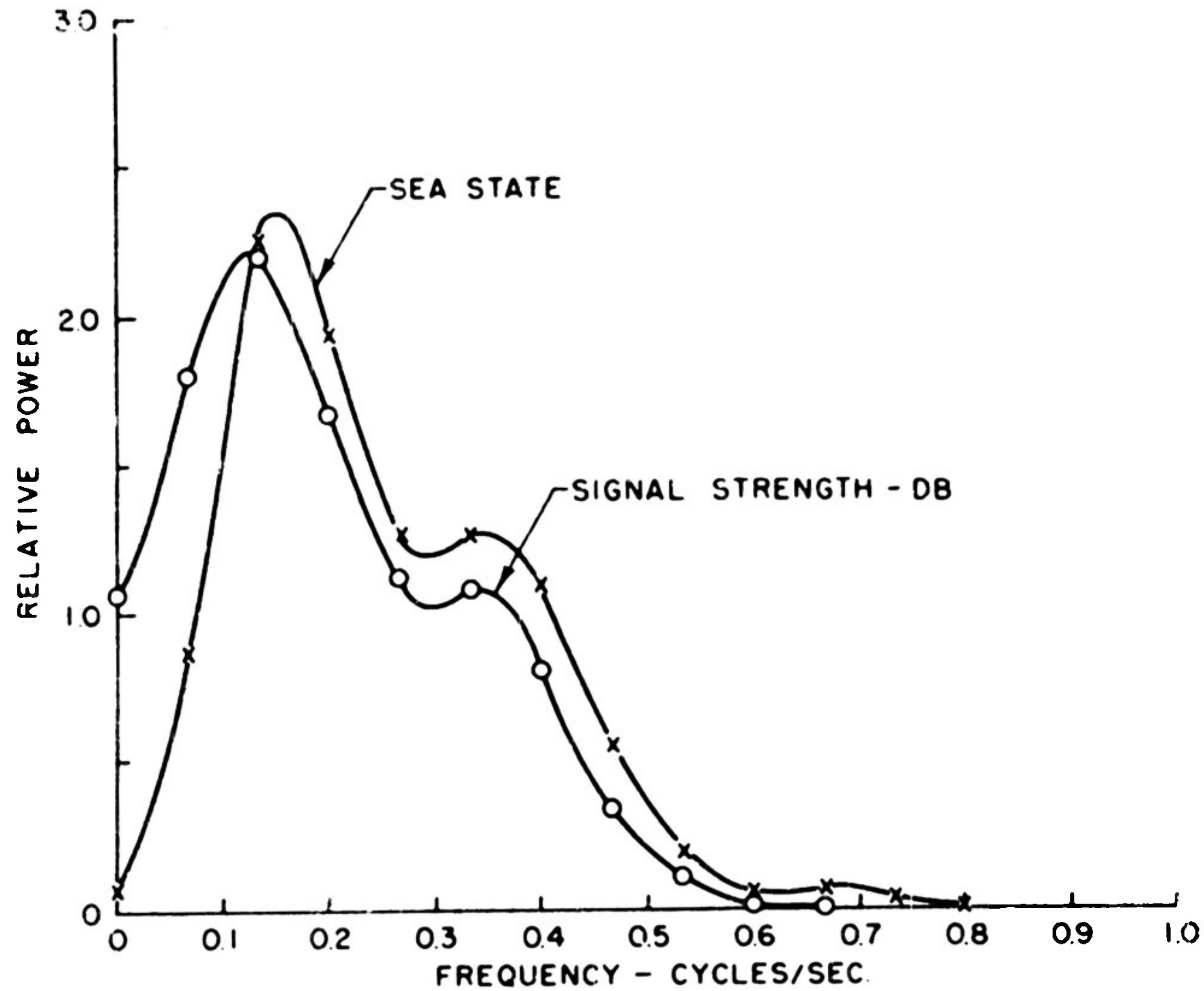


FIG. 4-NORMALIZED POWER SPECTRUM ESTIMATES (TUKEY METHOD) FOR 3.2 CM. RADIO SIGNALS AND SIMULTANEOUS SEA STATE – AUGUST 6, 1952, TIME: 0758

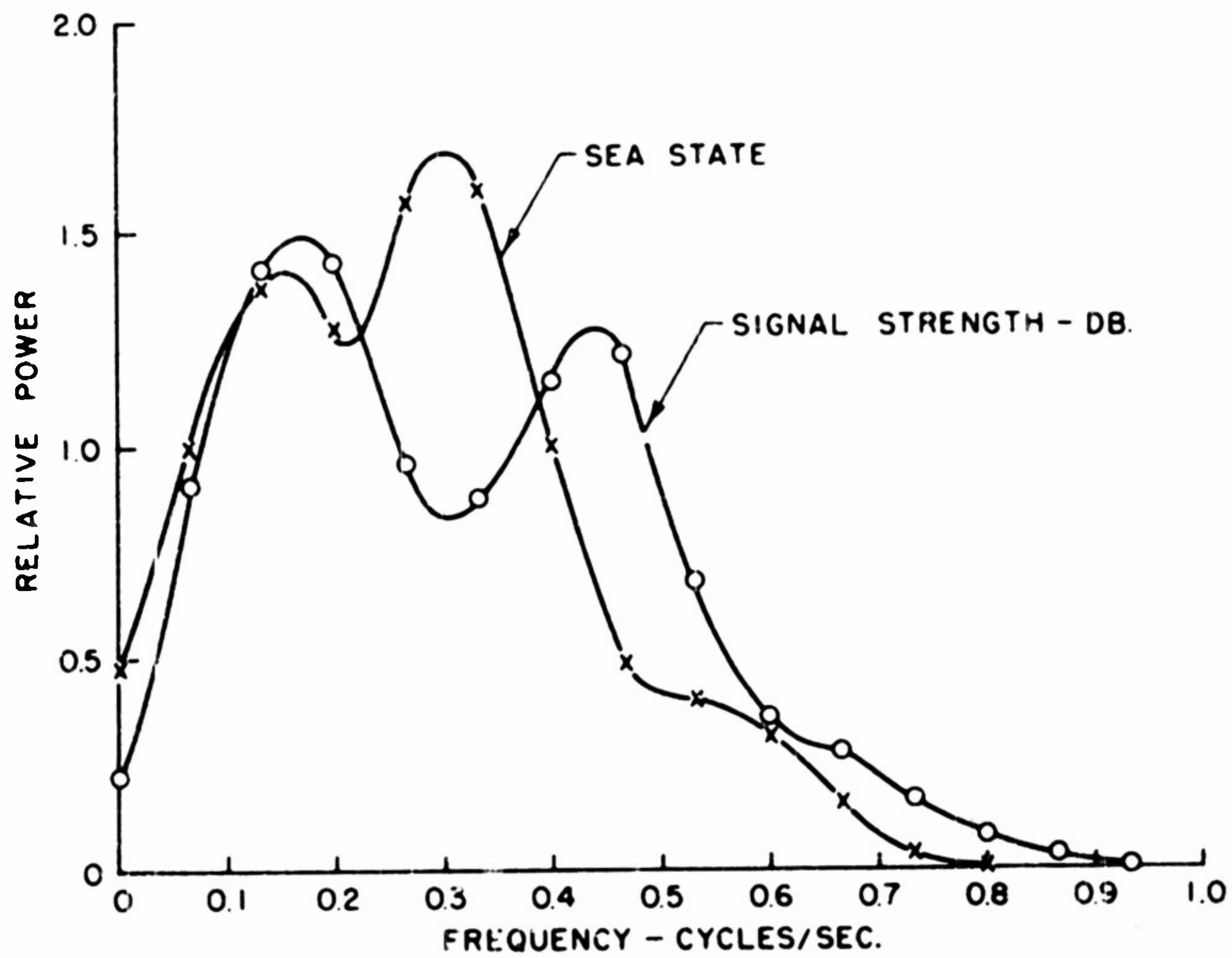


FIG. 5-NORMALIZED POWER SPECTRUM ESTIMATES (TUKEY METHOD) FOR 0.86 CM. RADIO SIGNALS AND SIMULTANEOUS SEA STATE - AUGUST 6, 1952, TIME: 0905

EERL 1351
4 - 53

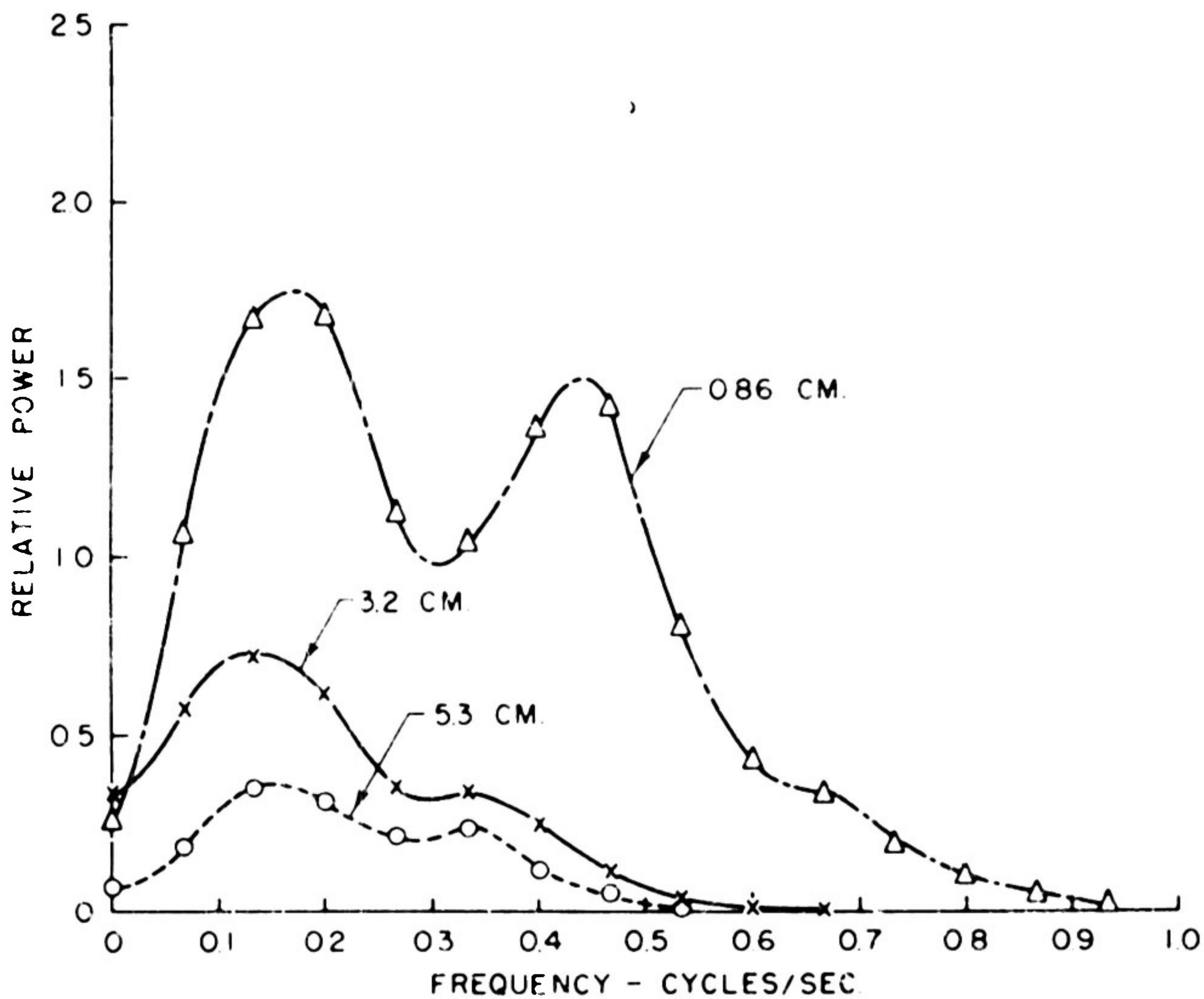


FIG. 6-RELATIVE POWER SPECTRUM ESTIMATES (TUKEY METHOD)
FOR 5.3 CM., 3.2 CM., AND 0.86 CM. RADIO SIGNALS
AUGUST 6, 1952.

EERL 1352
4 - 53

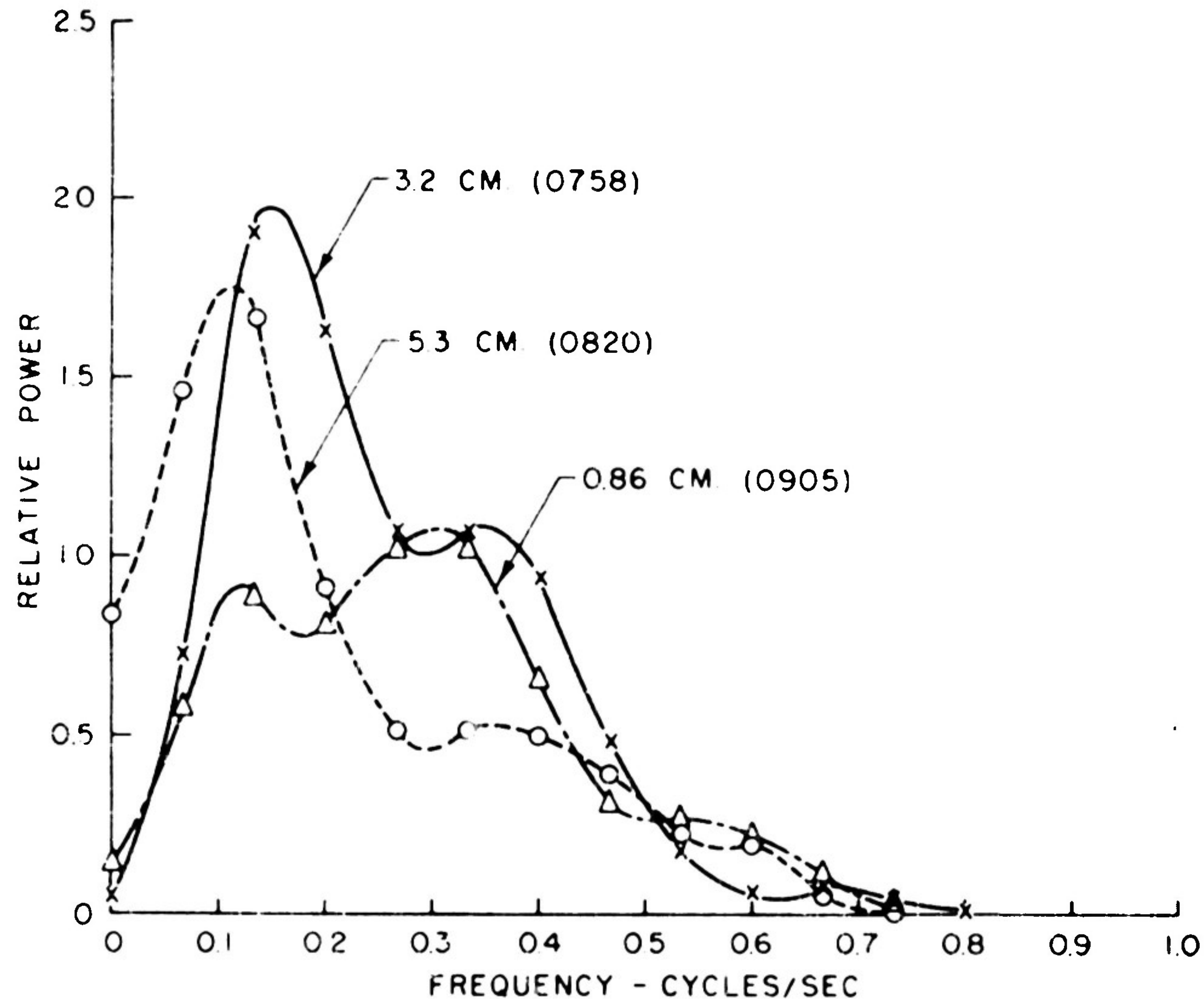


FIG. 7-RELATIVE POWER SPECTRUM ESTIMATES (TUKEY METHOD) FOR SEA STATE CORRESPONDING TO 5.3 CM., 3.2 CM AND 0.86 CM. RADIO SIGNALS - AUGUST 6, 1952.

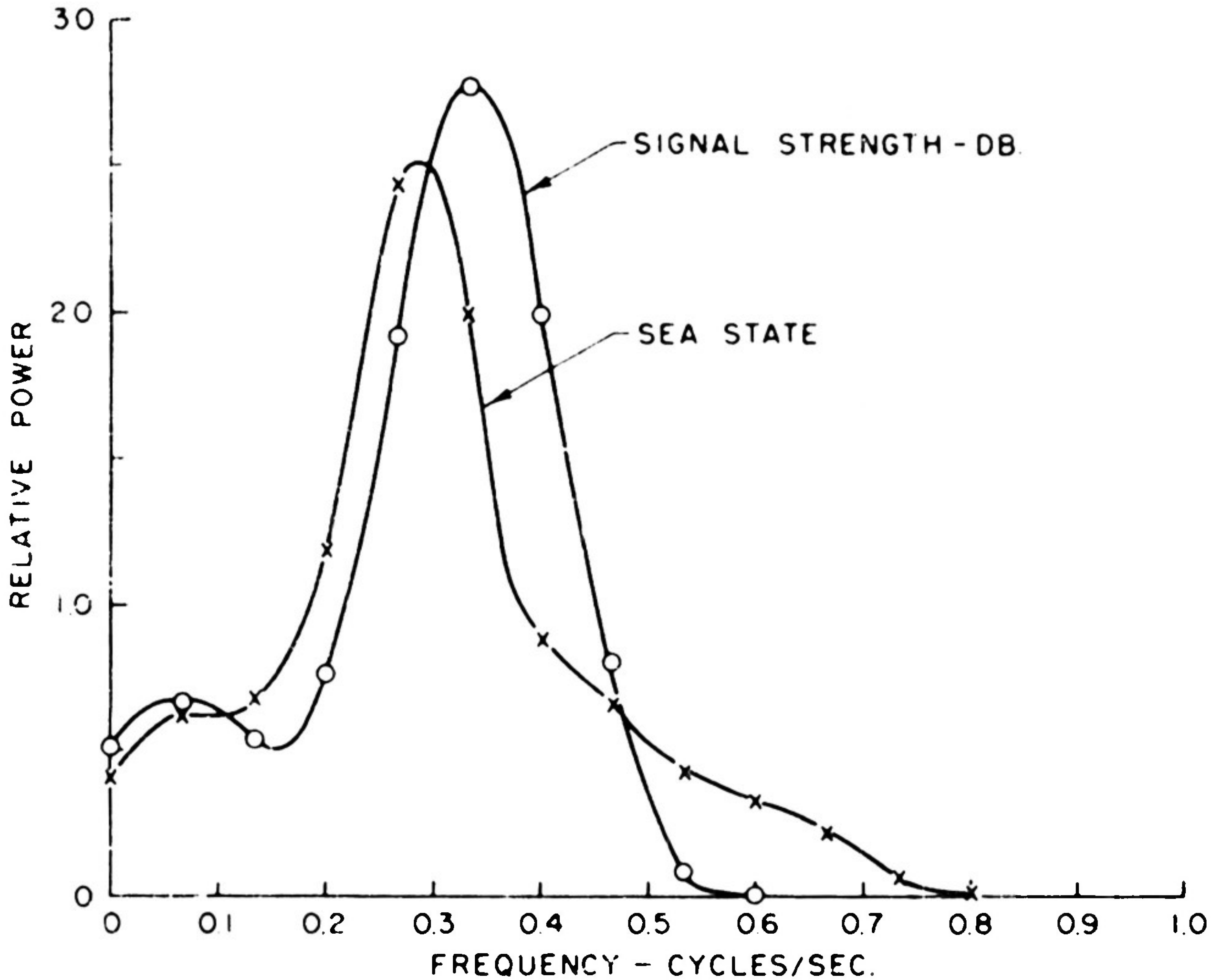


FIG. 8 - NORMALIZED POWER SPECTRUM ESTIMATES (TUKEY METHOD) FOR 9.0 CM. RADIO SIGNALS AND SIMULTANEOUS SEA STATE - AUGUST 7, 1952, TIME: 1125

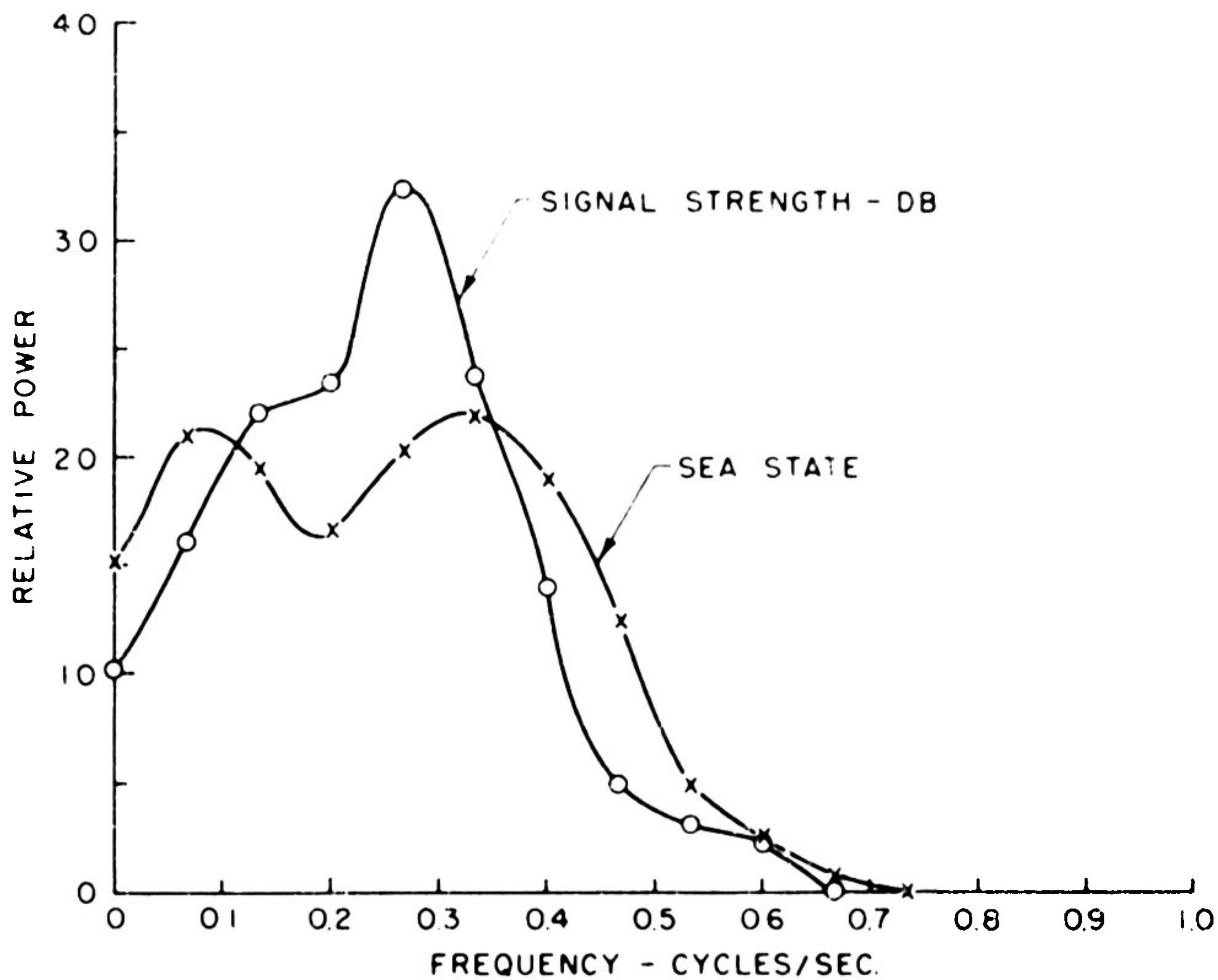


FIG. 9-NORMALIZED POWER ESTIMATES (TUKEY METHOD) FOR 5.3 CM. RADIO SIGNALS AND SIMULTANEOUS SEA STATE - AUGUST 7, 1952, TIME: 1045

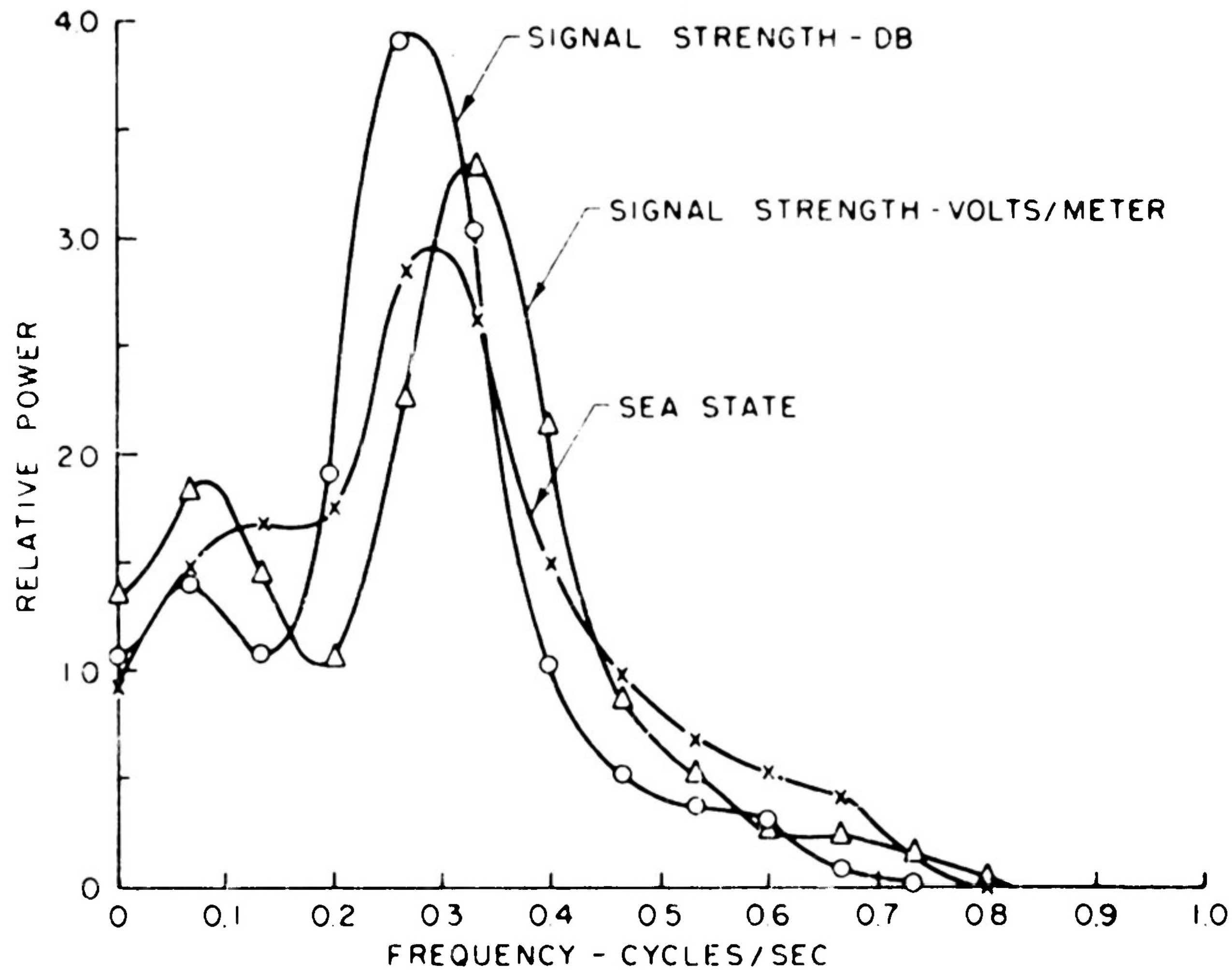


FIG. 10-NORMALIZED POWER SPECTRUM ESTIMATES (TUKEY METHOD) FOR 3.2 CM RADIO SIGNALS ON DB. AND VOLTAGE BASIS AND SIMULTANEOUS SEA STATE - AUGUST 7, 1952, TIME - 1100.

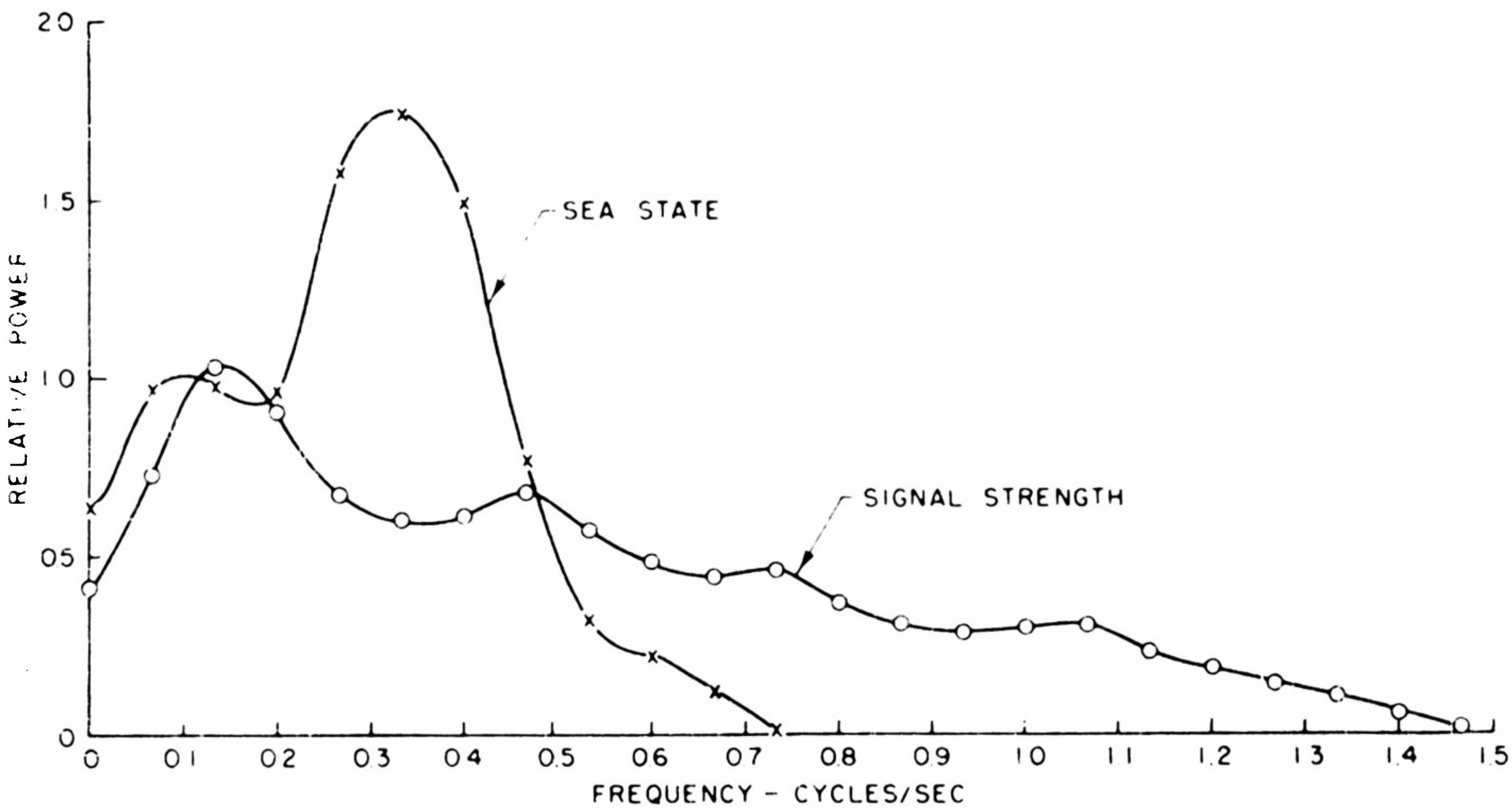


FIG. 11 - NORMALIZED POWER SPECTRUM ESTIMATES (TUKEY METHOD)
FOR 0.86 CM. RADIO SIGNALS AND SIMULTANEOUS
SEA STATE - AUGUST 7, 1952, TIME: 1010

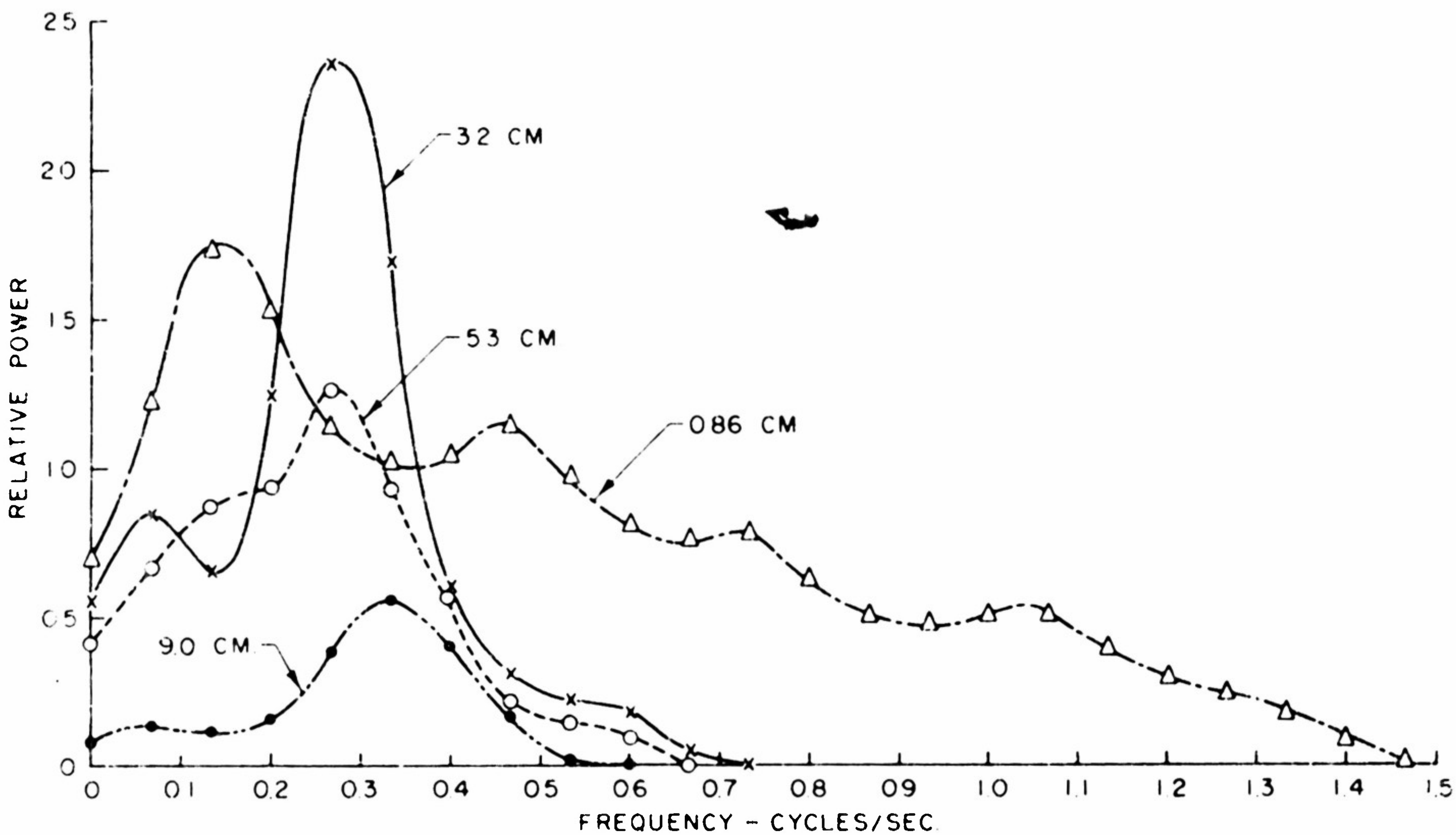


FIG. 12 - RELATIVE POWER SPECTRUM ESTIMATES (TUKEY METHOD) FOR 90 CM, 53 CM, 32 CM, AND 0.86 CM RADIO SIGNALS. AUGUST 7, 1952

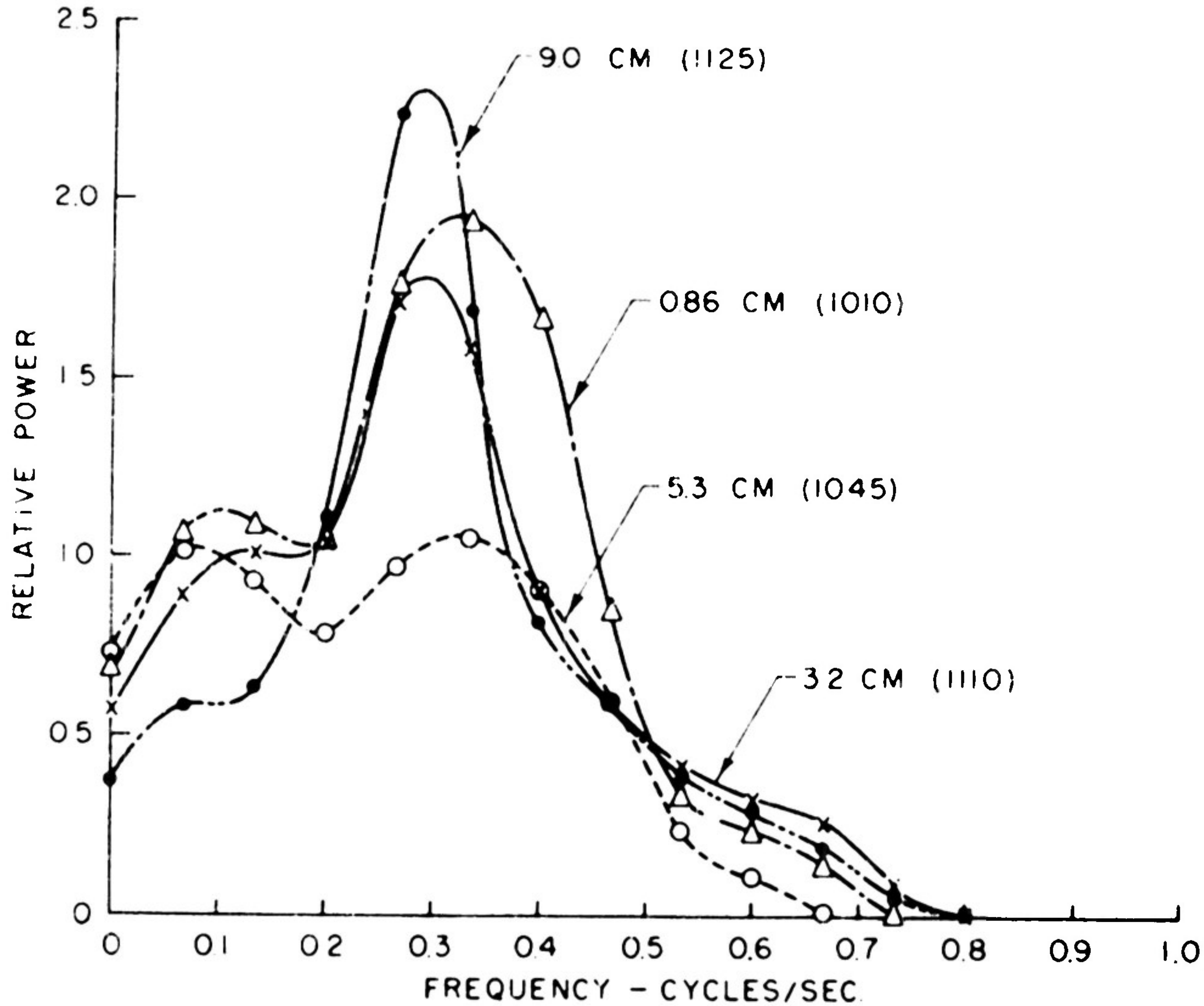


FIG. 13 - RELATIVE POWER SPECTRUM ESTIMATES (TUKEY METHOD) FOR SEA STATE CORRESPONDING TO 9.0 CM., 5.3 CM., 3.2 CM., AND 0.86 CM RADIO SIGNALS - AUGUST 7, 1952

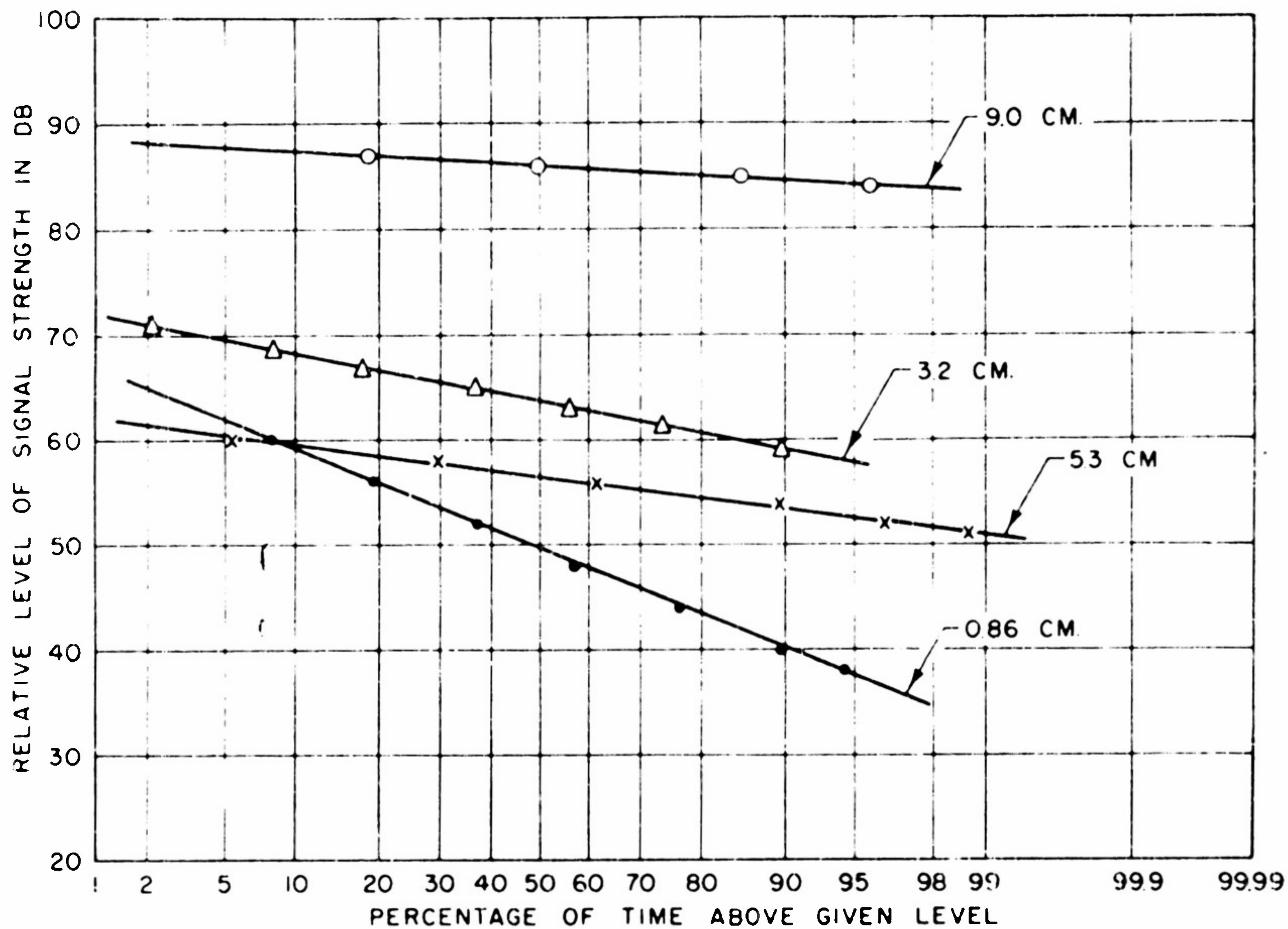


FIG 14 - ARITHMETIC PROBABILITY PLOT FOR RADIO SIGNAL STRENGTH IN DB. FOR WAVELENGTHS OF 9.0, 5.3, 3.2, AND 0.86 CM. ON AUGUST 7, 1952

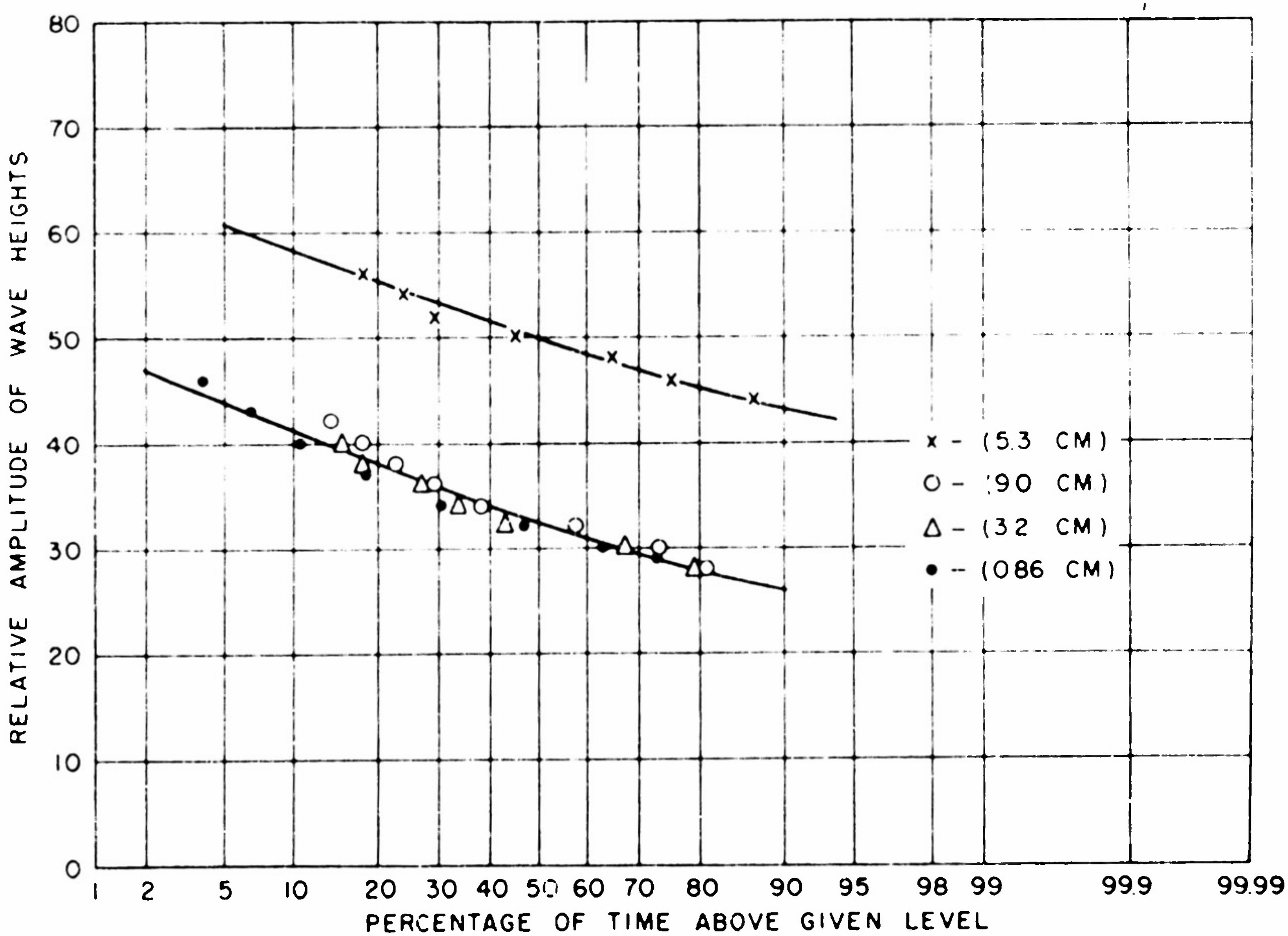


FIG. 15 - ARITHMETIC PROBABILITY PLOT OF SEA STATE DATA CORRESPONDING TO RADIO SIGNALS AT 9.0, 5.3, 3.2, AND 0.86 CM. ON AUGUST 7, 1952.

VIII. CONCLUSIONS

In view of the many uncertainties involved and because of the complexity of the phenomena of the reflection of radio signals from a rough surface any definite conclusions based on such a short time interval and so few samples would be doubtful. It is felt that the principal value of the information is that it brings out points which seem to warrant further investigation both from the experimental and the analytical viewpoint.

The following points of interest may be noted:

1. The correspondence between the power spectra for signal strength and sea state appears to be better for the longer wavelengths.

2. The power spectra for the 0.86-cm signal strength shows little similarity to the sea state and contains higher frequencies than are present in the sea state.

3. The rms value of the signal strength fluctuations is in nearly every case larger for the shorter wavelengths.

4. Power spectra for the sea state data shows considerable variation between runs on the same day as well as runs taken on different days. This would be expected since the weather conditions were quite variable during the period of the tests.

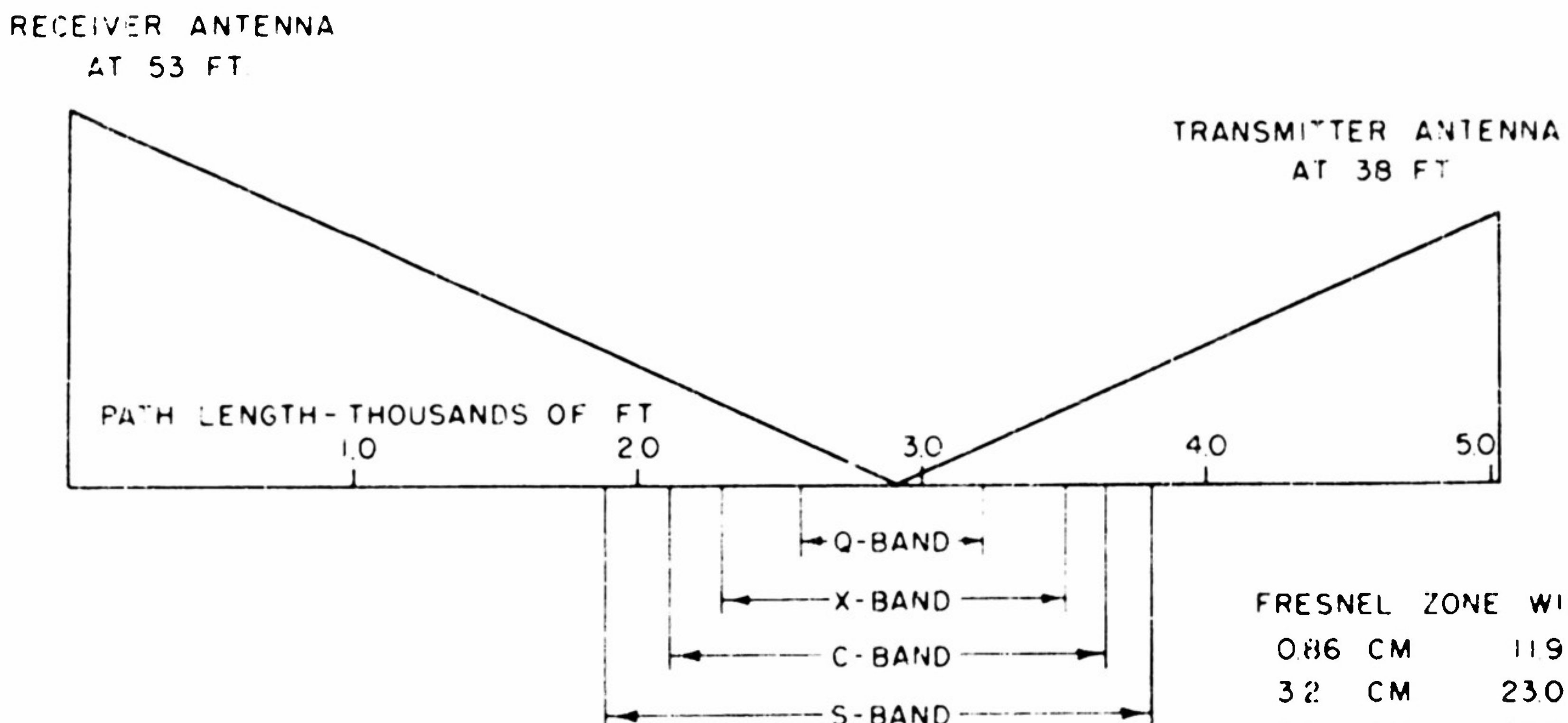
5. The signal strength distribution is approximately log-normal, but the sea state data shows some deviation from either a normal or a log-normal distribution.

As a means of gauging the comparative roughness of the sea surface to different signal wavelengths for the water waves, the number of water wave peaks in the first Fresnell zone (see Figure 16) for each radio signal was plotted against the frequency of the water waves (Figure 17). The number of wave crests was plotted from the equation for water-wave length for wind generated [4] in deep water,

$$L = 5.12\ T^2$$

where L is the length of the wave in feet and T is the period of the wave. Figure 17 shows that for shorter radio signal wavelengths there are fewer wave crests at a particular water-wave frequency than for longer-wave-length radio signals. If the fluctuations of the radio signals were caused by the horizonal movement of the waves, then it would be expected that the shorter wavelength radio signals would be affected by the water waves than longer wavelength signals. The power spectra of the signals behaved in this manner.

To investigate the effect of the vertical movement of the water causing the reflected signal to pass in and out of phase with the direct signals, the change in water height required for such a phase change was determined for the different signal frequencies. These varied from 0.5 ft for the 0.86-cm signal to four ft for the 9.0 signal. From this we would expect the shorter-wave-length radio signals to be more affected by the vertical movement of the water than the longer-wave-length signals.



FRESNEL ZONE WIDTHS

0.86 CM	11.9 Ft
3.2 CM	23.0 Ft
5.3 CM	29.5 Ft
9.0 CM	38.5 Ft

FIG. 16 - FIRST FRESNEL ZONES FOR RADIO SIGNALS

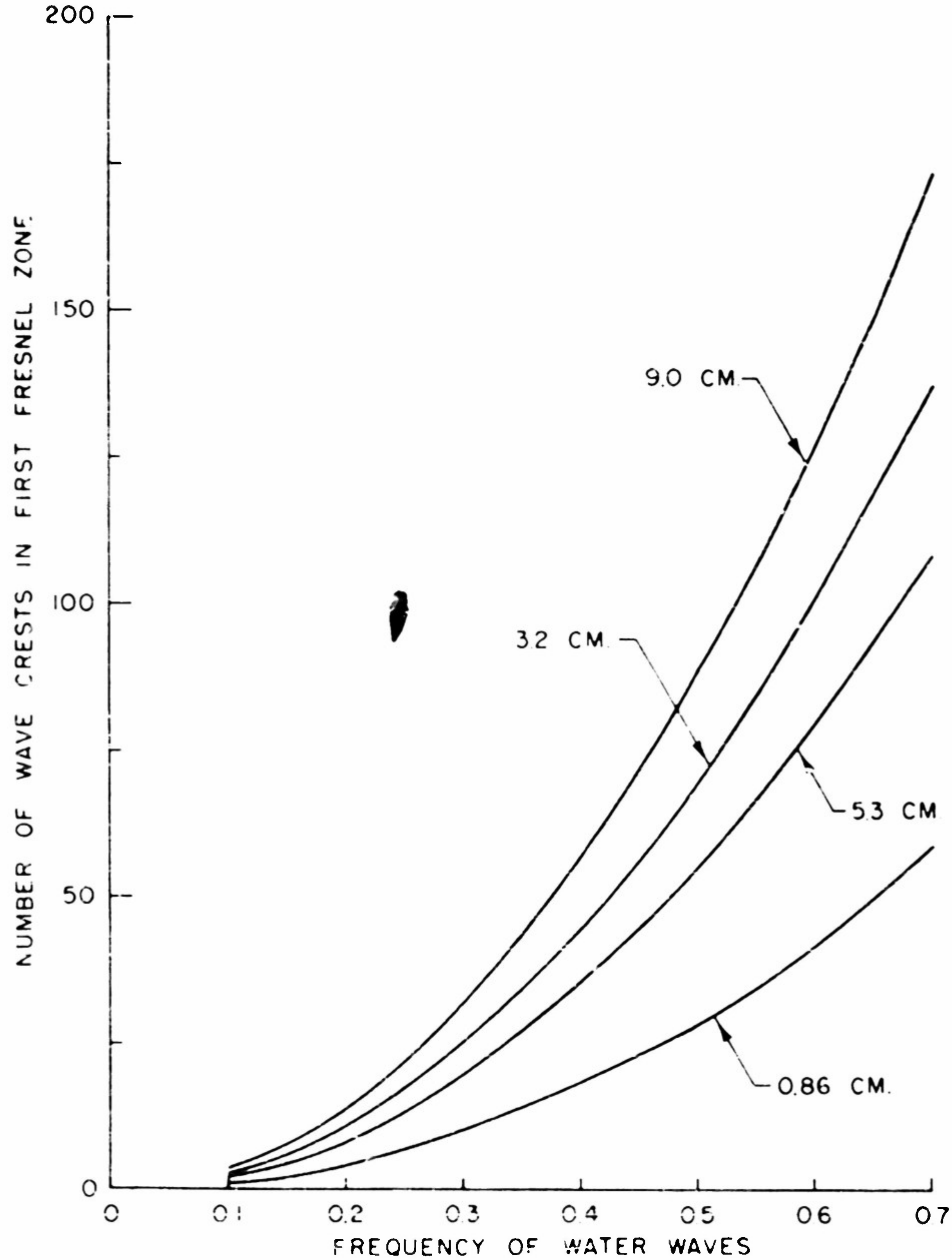


FIG 17-NUMBER OF WAVE CRESTS IN FIRST FRESNEL ZONE AS A FUNCTION OF WATER WAVE FREQUENCY.

APPENDIX

Analysis Techniques

The method of obtaining the power spectrum estimates may be summarized by the following steps:

1. Autocorrelations were made on the original data by a correlation computer using approximately 4000 sampling points for 40 time lags. In each case duplicate autocorrelations were made to provide a check on the computer operation. It may be stated, however, that the reliability and accuracy of the computer has been well established in previous computations.

2. From a method outlined by Tukey [2], the "raw estimates" of the power spectrum were calculated from the autocorrelation by performing the Fourier cosine transformation,

$$S_h = \frac{1}{m}\left[A_0 + 2\sum_{n=1}^{m-1} A_n \cos\frac{\pi n h}{m} + A_m \cos \pi h\right] (h = 0,1,\ldots m) \tag{1}$$

where A_n is the autocorrelation coefficient for nth lag. Dr. Tukey has pointed out that these "raw estimates" are subject to serious errors because of the mathematical approximations involved and the propagation of substantial statistical fluctuations resulting from the sampling process throughout the successive steps in the utation.

3. These "raw estimates" may then be smoothed by applying the correct factors which determine the interrelation between adjacent values. The "smoothed power spectrum estimates" are thus obtained by calculating

$$U_h = 0.23\, S_{h-1} + 0.54\, S_h + 0.23\, S_{h+1} \tag{2}$$

It is these values which are then plotted as a function of frequency in Figures 3 to 13. This results in the total power between frequencies $(h-1)\frac{\Pi}{m}$ and $(h+1)\frac{\Pi}{m}$, except for the end points.

4. Dr. Tukey has further shown that under the assumption of an essentially Gaussian distribution for the original data, the values of U_n are distributed according to a chi-squared distribution with f degrees of freedom,

$$f = \frac{N - m/4}{m/2} \tag{3}$$

where N = total number of sample points

m = total number of lags.

Figure 18 shows the behavior of a chi-squared distribution as a function of the number of degrees of freedom. Thus for 50 degrees of freedom the estimate will lie between 0.74 and 1.45 of the true value 90% of the time. The accuracy of the estimate increases with the number of degrees of freedom. This increase in accuracy is rather gradual above 50 degrees of freedom but drops off abruptly as the number is decreased.

The number of degrees of freedom for the curves in this report is approximately 40. Figure 19 illustrates 90% limits for 40 degrees of freedom for the 5.3-cm radio data on 6 August, 1952. Included on this curve are two points on the 90% limit curves for 100 degrees of freedom. This would correspond to about 2½ times as much data for the same resolution of points on the power spectrum estimate.

It should be noted that for a fixed length of data more resolution could be obtained only at the expense of degrees of freedom and, hence, accuracy. In the examples in this report approximately 40 degrees of freedom were chosen to maintain a given probably accuracy. This, of course, limited the bandwidth or resolution to the values shown. It would be desirable to increase both the resolution and degrees of freedom, but this could only be done by taking longer samples of data. It may be stated that much longer data of a similar nature is now available and is to be analyzed in the near future.

It is also very important that the number of samples taken from the autocorrelation to obtain the "raw estimates" of the power spectrum be sufficient to detect the highest frequency components in the original data. If this is not done, power from the higher-frequency components will be aliased into (or added into) the true value at the band desired. In other words, power actually present in the record at higher frequencies will appear as power in the bandwidth at lower frequencies. In the present examples, the choice of sampling interval was chosen to detect frequencies up to about 10 cycles per second. This is far more than necessary, since the response of the recording instruments is limited at frequencies much above 1 cycle per second. This choice, however, should not have limited the accuracy, but, instead, simply required more computation than may have been necessary.

As was pointed out earlier, this approximation of probable accuracy is based upon a Gaussian distribution of the original data. The smallness of the deviation from a straight line variation on Figures 14 and 15 shows that this was approximately true. It is of interest to note that the signal strength expressed in decibels is more nearly Gaussian than when expressed in volts. It was found that when the water data are expressed in decibels, the result was no more Gaussianly distributed than when expressed as the original difference quantity.

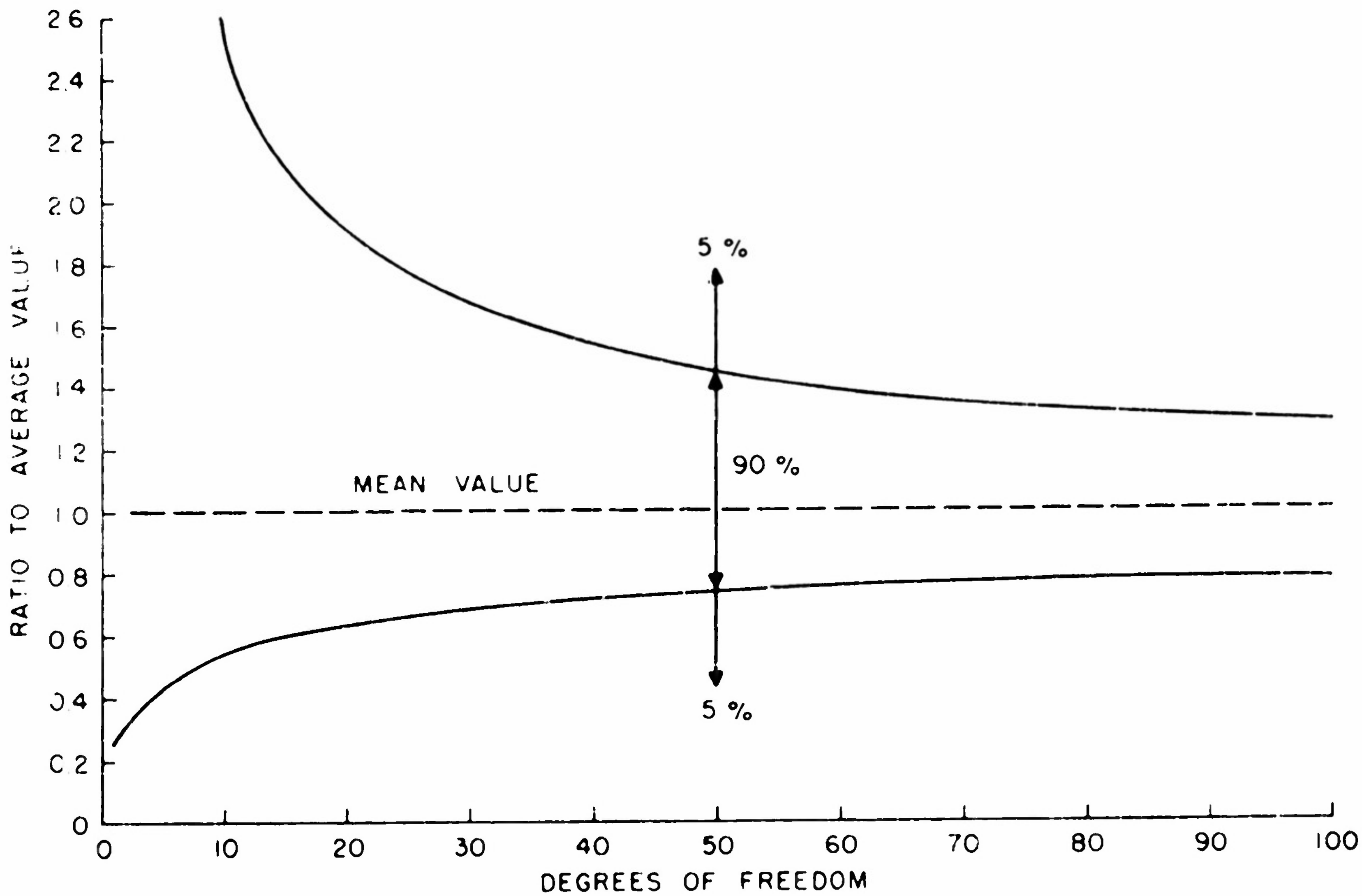


FIG 18-BEHAVIOR OF CHI-SQUARE

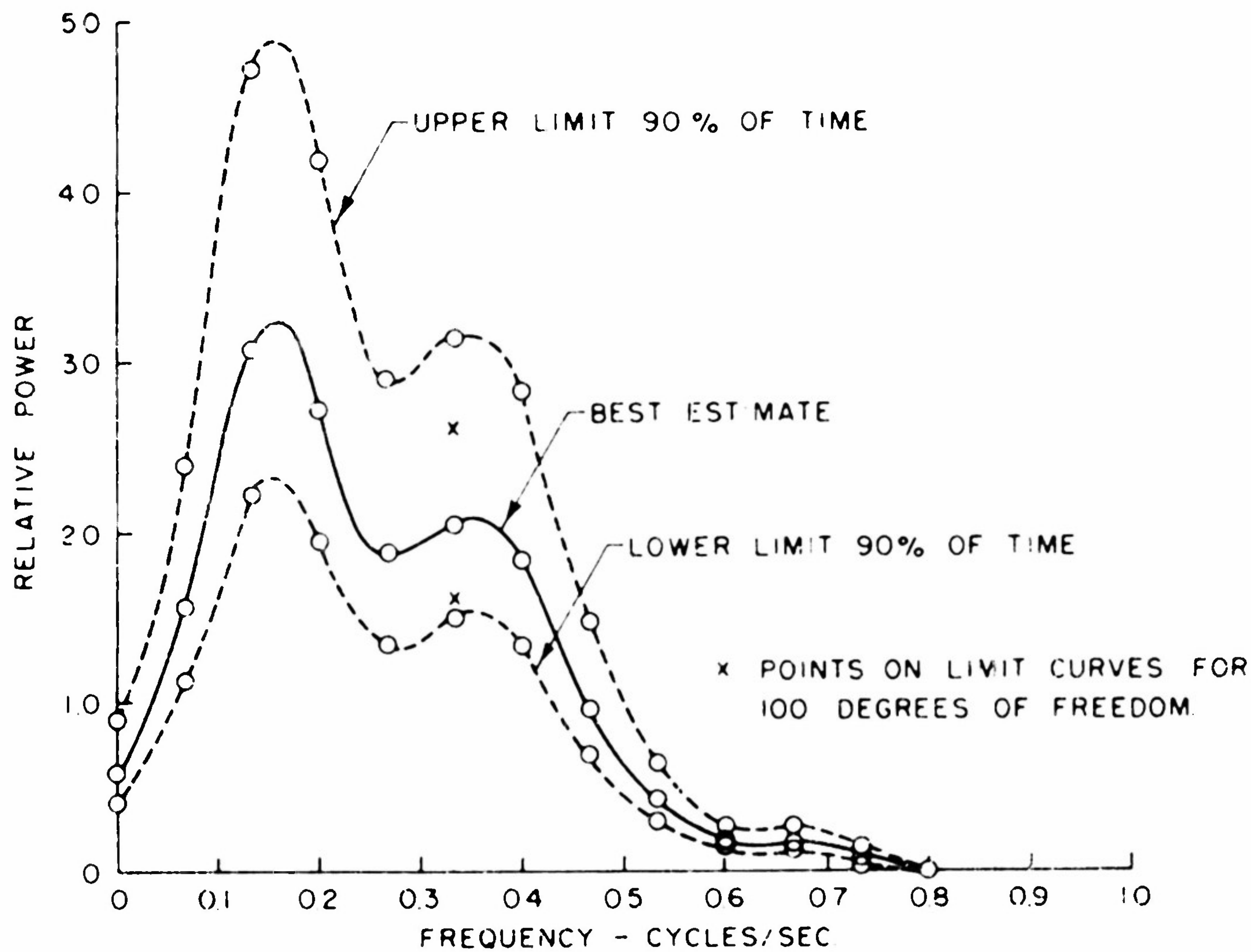


FIG. 19 - POWER SPECTRUM ESTIMATE FOR 5.3 CM. RADIO SIGNAL ON AUGUST 6, 1952 SHOWING LIMITS OF POSSIBLE DEPARTURE FROM TRUE VALUE 90 PER CENT OF TIME FOR 40 DEGREES OF FREEDOM.

REFERENCES

1. LaGrone, A. H. and C. W. Tolbert, "Reflection Studies of Millimeter and Centimeter Radio Waves for Gulf of Mexico Paths," Electrical Engineering Research Laboratory, The University of Texas, Report No. 64, October, 1952.

2. Tukey, J. W., "The Sampling Theory of Power Spectrum Estimates in Symposium on Applications of Autocorrelation Analysis to Physical Problems," Woods Hole, Massachusetts, June 13-14, 1949.

3. Brooks, F. E., Jr. and H. W. Smith, "A Computer for Correlation Functions," Electrical Engineering Research Laboratory, The University of Texas, Report No. 50, May 1951.

4. "Techniques for Forecasting Wind Waves and Swell," H. O. Pub. No. 604 U. S. Navy Hydrographic Office, Washington, D. C.

DISTRIBUTION LIST

Nonr 375(01)

Addressee	Attention, Code	No. of Copies
Chief of Naval Research Office of Naval Research Washington 25, D. C.	Electronics Branch Code 427	2
Director, Naval Research Laboratory Washington 25, D. C.	Technical Information Officer	9
Chief of Naval Research Office of Naval Research Washington 25, D. C.	Naval Sciences Division Code 460	1
Chief of Naval Research Office of Naval Research Washington 25, D. C.	Geophysics Branch Code 416	1
Director, Naval Research Laboratory Washington, D. C.	186-A 1360	3 1
Chief, Bureau of Ships Washington 25, D. C.	910 930	1 1
Chief, Bureau of Aeronautics Washington 25, D. C.	EL50 EL90	1 1
Chief, Bureau of Ordnance Washington, D. C.	Re4f	1
Chief of Naval Operations Washington 25, D. C.	Op20Q2 Op55F OP413	1 1 1
Office of Naval Research Brooklyn, New York		1
Office of Naval Research Chicago 11, Illinois	Commanding Officer	2
Office of Naval Research Pasadena 1, California	Commanding Officer	1
Office of Naval Research San Francisco 9, California	Commanding Officer	1
Assistant Naval Attache for Research New York, New York		1
Naval Ordnance Laboratory Washington, D. C.		1

Addressee	Attention, Code	No. of Copies
Commanding Officer U. S. Naval Ordnance Test Station Inyoker, California		1
U. S. Navy Electronics Laboratory San Diego 52, California	Dr. J. E. Smyth	1
Librarian U. S. Naval Postgraduate School Monterey, California		1
Commandant, Coast Guard Washington, D. C.	EEE	1
Office of the Chief Signal Officer Washington, D. C.	sigcl-2 SIGET	1 1
Commanding Officer Fort Monmouth, New Jersey		1
Chief, Electronics Subdivision Wright Field, Dayton, Ohio		1
Commanding Officer 3151st Electronics Group Griffiss Air Force Base Rome, New York	ENBPF	1
Commanding Officer AMC Cambridge Field Station Cambridge, Massachusetts		1
Commanding General Hq U. S. Air Force Washington, D. C.	AC-AS/4:AFDRE	1
Research and Development Board Washington, D. C.	Deputy Executive Director Committee on Geophysical Sciences	1
Central Radio Propagation Laboratory Washington, D. C.		
The Office Librarian Harvard University Cambridge 38, Massachusetts		1
Cornell University Ithaca, New York	Prof. C. R. Burrows	1
Stanford University Palo Alto, California	Dean F. E. Terman	1
British Commonwealth Scientific Office Washington, D. C.		2

Addressee	Attention, Code	No. of Copies
Radio Corporation of America New York, New York	Dr. H. H. Beverage	1
Bell Telephone Laboratories Red Bank, New Jersey	Dr. H. T. Friis	1
General Electric Company Schenectady, New York	PROJECT HERMES	1
U. S. Weather Bureau Washington, D. C.	Director, Radar Engineering	1
University of Illinois Urbana, Illinois	Prof. E. C. Jordan	1
Georgia School of Technology Atlanta, Georgia	Dr. J. E. Boyd	1
University of Michigan Ann Arbor, Michigan	Dr. S. S. Attwood	1
Naval Attache Australian Embassy Washington, D. C.		1
Research Laboratory of Electronics Massachusetts Institute of Technology Cambridge, Massachusetts	Prof. H. Zimmerman	1
Federal Communications Commission Washington 25, D. C.	Technical Information Division	1
U. S. Navy Underwater Sound Laboratory New London, Connecticut		1
University of California Berkeley 4, California	Prof. T. C. McFarland	1
Commanding Officer Naval Air Development Station Johnsville, Pennsylvania	AAEL	1
National Bureau of Standards Boulder Laboratories, Radio Division Boulder, Colorado		2
Commanding General Rome Air Development Center Griffiss Air Force Base Rome, New York	Technical Library RCRTL	1
Johns Hopkins University Applied Physics Laboratory 8621 Georgia Avenue Silver Springs, Maryland	Arthur G. Norris	1

Addresses	Attention, Code	No. of Copies
Project Lincoln, MIT P. O. Box 390 Cambridge 39, Massachusetts	Dr. T. J. Carroll	1